NUMERICAL STUDY OF TWO-DIMENSIONAL THERMOVIBRATIONAL CONVECTION IN RECTANGULAR CAVITIES

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Two-dimensional thermovibrational convection in rectangular cavities under the condition of weightlessness is studied. The problem is based on the system of equations of the mean fields of velocity, pressure, and temperature. A pseudospectral Chebyshev collocation method is used. The case of rectangular cavities (a layer of finite length) is considered subject to high-frequency transversal vibrations and a longitudinal temperature gradient. In the case of a square cavity the instability of the main flow exists, and the bifurcation to other symmetry takes place. The same behavior is observed when the cavity is elongated in the direction of the temperature gradient. It is shown that the intensity of the thermovibrational convective flow decreases, in general, while the aspect ratio increases in accordance with linear stability theory, in which it was proven that, in the limiting case of an infinitely long layer subject to a longitudinal temperature gradient and a transversal axis of vibrations, the absolute stability of the quasi-equilibrium state takes place.

INTRODUCTION

In a closed cavity filled with fluid in the presence of inhomogeneity of temperature and high-frequency vibrations, a regular mean flow is generated even in the case of weightlessness, i.e., when the static gravity field is absent (the phenomenon of thermovibrational convection; see Refs. [1, 2]). The description of

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NOMENCLATURE

a	width of the cavity	ϵ	dimensional parameter of vibration, $[= \frac{1}{2}(\beta b \Omega)^2]$
b	displacement amplitude	Θ	temperature at $x = 0$
L	length of the cavity	ν	kinematic viscosity
$\mathbf{n}$	unit vector along the axis of vibrations	ρ	density
Nu	mean Nusselt number	χ	thermal diffusivity
p	pressure	ψ	stream function
Pr	Prandtl number	ψ_{max}	maximum value of the stream function
Ra_v	vibrational Rayleigh number	Ω	angular frequency of vibration
t	time		
T	temperature		
$\mathbf{u}$	velocity vector		
$\mathbf{w}$	solenoidal part of $T\mathbf{n}$	**Superscript**	
x, y	coordinates		
β	coefficient of thermal expansion	n	time level

thermovibrational flows in the limiting case of high frequency and small amplitude of vibration may be effectively obtained in the frame of the averaging method, which leads to the system of equations for average (mean) fields of velocity, pressure, and temperature. The mechanical quasi-equilibrium state is possible (i.e., the state at which the mean velocity is absent but the oscillatory component is, in general, not zero). Some examples of the mechanical quasi-equilibrium state with the results of linear stability analysis are presented in Refs. [1–3]. Perhaps the simplest case of mechanical quasi-equilibrium is an infinitely long plane layer in the presence of vibrations along the longitudinal axis and a transversal temperature gradient. This state of quasi-equilibrium becomes unstable when a nondimensional parameter, the vibrational analog of the Rayleigh number Ra_v, exceeds some critical value, which depends on the boundary conditions on the parallel planes bounding the fluid layer [1, 2]. The periodic system of convective cells appears at the threshold of instability. This is the vibrational analog of the classical Rayleigh-Benard problem of convective instability of the plane horizontal fluid layer when heated from below.

In the opposite limiting case of an infinitely long plane layer in the presence of a longitudinal temperature gradient and harmonic vibrations along the transversal axis, the quasi-equilibrium state is possible too, but the stability shows this quasi-equilibrium configuration as absolutely stable [3].

In the case of the layer of finite length (rectangular cavity) the quasi-equilibrium state is not possible, and the thermovibrational convective flow sets in at infinitely small values of temperature difference. When the intensity of the thermovibrational flow is large enough, the nonlinear approach is necessary to describe the situation. For the case of rectangular cavities with a longitudinal axis of vibration and a transversal temperature gradient, the nonlinear approach has been performed using a finite difference method [4]. The structures of the flows were studied. It has been demonstrated that the transition from the basic to a multicellular flow sets in at a critical value of the Ra_v. When the aspect ratio equals 8 or more, the character of the bifurcation is practically the same as in the limiting case of an infinitely long layer. We do not give here the exhaustive review of the thermovibrational convection nonlinear solutions. Additional bibliography can be found in Ref. [5].

In the present work we consider the opposite case of a thermovibrational convective phenomenon in a rectangular cavity, namely, we consider the configuration when the temperature gradient is longitudinal and the axis of vibration is transversal. A pseudospectral numerical method has been used. This method employs the "Darcy-Euler" solver developed in Ref. [6] and the Stokes solver developed in Ref. [7]. The Helmoltz and Poisson equations obtained are solved using a Chebyshev collocation method with the points of Gauss-Lobatto zeros, thus allowing a highly accurate solution for problems involving boundary layers (see Ref. [9]).

We show that in the cases where aspect ratios do not equal 1, the typical picture of the bifurcation connected to the reconstruction of the flow structure takes place as in the case of a square cavity. When the aspect ratio is large enough, the intensity of the thermovibrational flow decreases. The limiting case of a very long cavity corresponds to the state of rest, and the heat transfer is the same as in the conductive regime. These results are in conformity with linear stability theory [3].

PROBLEM DESCRIPTION

Let us consider a two-dimensional rectangular cavity filled with fluid (Figure 1). All the boundaries are rigid. The parts of the boundary at $x = 0$ and $x = L$ are isothermal and maintained at constant temperatures $T = \Theta$ and $T = 0$, respectively; the parts of the boundary at $y = 0$ and $y = a$ are thermally insulated, so $\partial T/\partial y = 0$. The whole system (the cavity with the fluid) is subject to linear and harmonic vibration along the y axis. Our consideration is based on the system of equations describing the behavior of the average (mean) fields of velocity, pressure, and temperature. This approach is valid within the framework of the averaging method in the limiting case of high frequency and small amplitude of vibration. Using the Boussinesq approximation, the governing equations are

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\nabla)\mathbf{u} = -\frac{\nabla p}{\rho} + \nu\nabla^2\mathbf{u} + \epsilon(\nabla\mathbf{w})(T\mathbf{n} - \mathbf{w}) \tag{1}$$

$$\frac{\partial T}{\partial t} + \mathbf{u}\nabla T = \chi\nabla^2 T \tag{2}$$

$$\nabla \cdot \mathbf{u} = 0 \tag{3}$$

$$\mathbf{w} + \nabla\phi = T\mathbf{n} \tag{4}$$

$$\nabla \cdot \mathbf{w} = 0 \tag{5}$$

Here $\mathbf{u}$ is the velocity, p is the pressure, T is the temperature, ν, χ, and β are the coefficients of kinematic viscosity, heat diffusivity, and thermal expansion, respectively, ρ is the reference value of density (constant), $\mathbf{n}$ is the unit vector along the

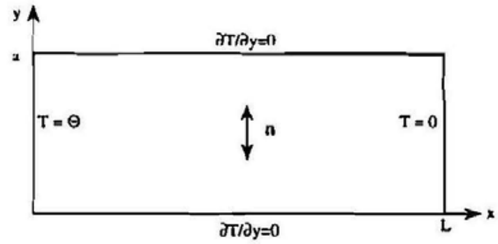


Figure 1. Geometrical configuration and axis of coordinates.

axis of vibration and $\epsilon = \frac{1}{2}(\beta b \Omega)^2$ is a dimensional parameter of vibration (b is the displacement amplitude and Ω is the angular frequency). The additional variable $\mathbf{w}$ is the solenoidal part of the field $T\mathbf{n}$; on the other hand, $\mathbf{w}$ is proportional to the amplitude of the oscillatory velocity component. We have already discussed the boundary conditions for $\mathbf{u}$ and T; with regard to $\mathbf{w}$, the condition of nonoverflow has to be posed. Thus we have

$$\begin{array}{lllll} x = 0 & (0 \le y \le a) & u_x = u_y = 0 & T = \Theta & w_x = 0 \\ x = L & (0 \le y \le a) & u_x = u_y = 0 & T = 0 & w_x = 0 \\ y = 0, a & (0 \le x \le L) & u_x = u_y = 0 & \dfrac{\partial T}{\partial y} = 0 & w_y = 0 \end{array} \qquad (6)$$

Let us now introduce the nondimensional form of the equation system and the boundary conditions with the help of the following characteristic quantities: a for length, χ/a for velocity, a^2/χ for time, Θ for temperature and $\mathbf{w}$ field, and $\rho\nu\chi/a^2$ for pressure. Thus we obtain the following nondimensionalized form of the system of equations:

$$\frac{1}{\Pr}\left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\nabla)\mathbf{u}\right) = -\nabla p + \nabla^2 \mathbf{u} + l^2 \,\mathrm{Ra}_v (\nabla \mathbf{w})(T\mathbf{n} - \mathbf{w}) \qquad (7)$$

$$\frac{\partial T}{\partial t} + \mathbf{u}\,\Delta T = \nabla^2 T \qquad (8)$$

$$\nabla \cdot \mathbf{u} = 0 \qquad (9)$$

$$\mathbf{w} + \nabla\phi = T\mathbf{n} \qquad (10)$$

$$\nabla \cdot \mathbf{w} = 0 \qquad (11)$$

With the boundary conditions

$$
\begin{array}{lllll}
x = 0 & (0 \le y \le 1) & u_x = u_y = 0 & T = 1 & w_x = 0 \\
x = l & (0 \le y \le 1) & u_x = u_y = 0 & T = 0 & w_x = 0 \\
y = 0, 1 & (0 \le x \le l) & u_x = u_y = 0 & \partial T/\partial y = 0 & w_y = 0
\end{array}
\tag{12}
$$

The nondimensional parameters of our problem are $\mathrm{Ra}_v = \frac{1}{2}[(\beta b \Omega A a^2)^2/\nu\chi]$, $\mathrm{Pr} = \nu/\chi$, $l = L/a$. Here Ra_v is the vibrational analog of the Rayleigh number, which is determined by the temperature gradient $A = \Theta/L$, Pr is Prandtl number, and l is the aspect ratio.

SOLUTION METHOD

The numerical method used here is based on the projection diffusion algorithm developed in Ref. [7] for solving the two-dimensional/three-dimensional unsteady incompressible Navier-Stokes equations. The temporal integration consists of a semi-implicit second-order finite differences approximation. The linear (viscous) terms are treated implicitly by the second-order Euler backward scheme, while a second-order explicit Adams-Bashforth scheme is employed to estimate the nonlinear (advective) parts. When applied to an advection diffusion equation such as

$$
\frac{\partial f}{\partial t} + (\mathbf{u}\nabla f) = \alpha \nabla^2 f \tag{13}
$$

the method reads

$$
\frac{\frac{3}{2}f^{n+1} - 2f^n + \frac{1}{2}f^{n-1}}{\Delta t} = \alpha \nabla^2 f^{n+1} - \left[2(\mathbf{u}\nabla f)^n - (\mathbf{u}\nabla f)^{n-1}\right] \tag{14}
$$

This last equation can be written in the form of the following Helmholtz equation:

$$
(\nabla^2 - h) f^{n+1} = s \tag{15}
$$

Here $h = 3/2\alpha\,\Delta t$ is the Helmholtz constant, and s is a scalar quantity containing all the terms known at time $t_n = n\,\Delta t$ (n is the time level and Δt is the time step). As one can see, the temporal integration transforms the system of equations, Eqs. (7)–(11), as follows: the energy equation, Eq. (8), becomes a simple Helmholtz one; Eqs. (10) and (11) with corresponding boundary conditions form a Darcy-Euler problem. A new method [6] based on a Uzawa operator is used to solve this problem. Finally, Eqs. (7) and (9) are transformed into a generalized Stokes problem modelized by the projection diffusion method of Ref. [7]. All the subproblems obtained are either Helmholtz or Laplace-like operators. A high-accuracy spectral method, namely, the Chebyshev collocation method, with the Gauss-Lobatto zeros as collocation points, is used in the spatial discretization of the

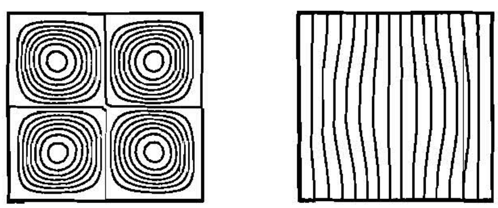
Helmholtz and Laplace-like operators. The well-known successive diagonalizations technique [8] is implemented to inverse these different operators. We have to mention that the Stokes and Darcy-Euler solvers are direct and guarantee a spectral accuracy solution with a free divergence for the **u** and **w** fields on the whole domain, including the boundaries.

RESULTS AND DISCUSSION

In our calculation presented hereafter, the Prandtl number was fixed at Pr = 1, the aspect ratios ranged from 1 to 20, and the values of the vibrational Rayleigh number were limited to $Ra_v = 2.5 \times 10^5$.

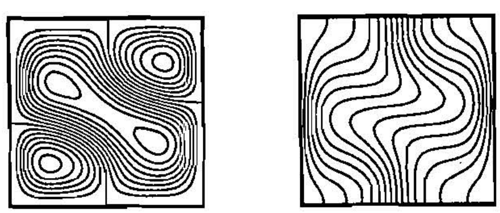
First, we consider the results for the case $l = 1$, which corresponds to the square cavity. Under the conditions described, the mechanical quasi-equilibrium state is not possible [2], so the steady regimes exist for infinitely low values of Ra_v. In the region of small Ra_v the thermovibrational convective flow is of a four-vortices structure (see Figure 2*a*). This flow is stable up to a critical value $Ra_v = Ra_{v*} \approx 8.5 \times 10^3$. When this value is exceeded, this steady regime becomes unstable, and a bifurcation takes place to another inversional symmetry (Figure 2*b*). The main branch corresponding to the four-vortices regime may be continued into the region of $Ra_v \geq Ra_{v*}$, but here, this regime is metastable (i.e., it is stable against disturbance of the same symmetry and unstable against disturbance of inversional symmetry).

In Figure 3 the bifurcation diagram is presented in Ra_v-Nu coordinates, where the Nusselt number Nu is the mean nondimensional heat flux through the



$Ra_v = 10^4$, $\psi_{max} = 0.3425$

(*a*)



$Ra_v = 10^5$, $\psi_{max} = 3.5790$

(*b*)

Figure 2. Streamlines and isotherms for two steady regimes ($l = 1$).

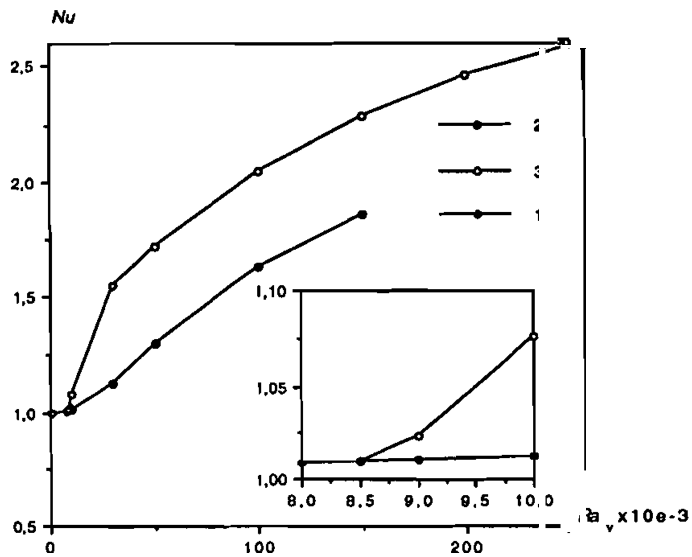


Figure 3. Bifurcation diagram in coordinates Ra_v-Nu (l = 1).

cavity. Curve 1 corresponds to the stable four-vortices regime, curve 2 corresponds to its metastable continuation, while curve 3 represents the steady inversional symmetric regime. The endpoint of curve 2 corresponds to the point of absolute instability of the four-vortices regime. The endpoint of curve 3 represents the transition to an oscillatory regime of flow. The results presented are in good agreement with those of Refs. [4, 5].

For the cases with other values of l, the situation is qualitatively close to that described for the case of l = 1 (see Figures 4 and 5 for cases l = 2 and l = 5,

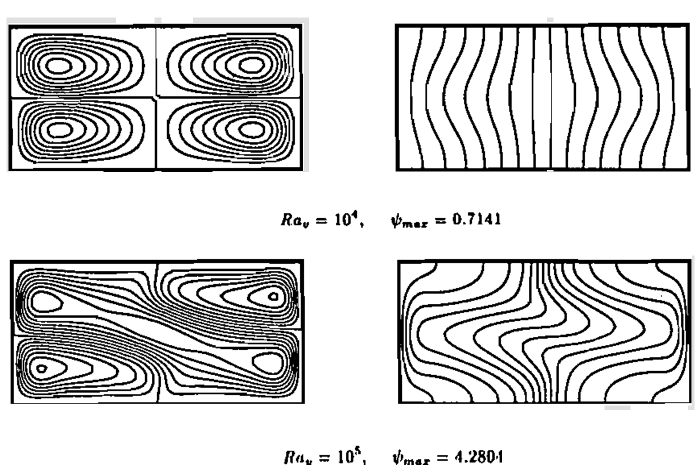


Figure 4. Streamlines and isotherms for two steady regimes (l = 2).

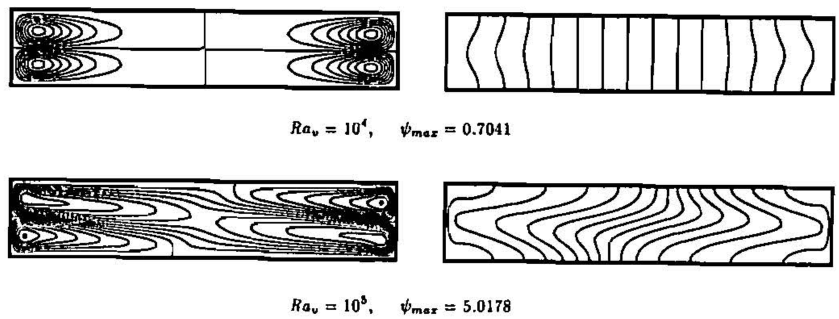


Figure 5. Streamlines and isotherms for two steady regimes (l = 5).

respectively). The same transition from the four-vortices regime to the inversional symmetric one has been observed. But, of course, all the characteristic points of the bifurcation diagram depend on the parameter l. In Table 1 the critical values of Ra_v for the first bifurcation are listed for different values of l. It is seen that Ra_{v*} increases monotonously as l increases.

Finally, we present in Figure 6 the characteristic of the vibrational convection intensity—the Nusselt number and the extremum value of the stream function—as a function of l at a fixed value of Ra_v ($Ra_v = 1 \times 10^5$). Note that for l large enough ($l \geq 10$), this value of Ra_v belongs to the region of $Ra_v \leq Ra_{v*}$, thus below the bifurcation point. It is interesting to note also that the dependence of Nu and ψ_{max} on l is not monotonous; there is a maximum intensity of the thermovi-

Table 1. Critical Rayleigh Number as a Function of the Aspect Ratio

	Value of l			
	1	2	5	10
Ra_{v*}	8.5×10^3	1.1×10^4	3.8×10^4	8.6×10^4

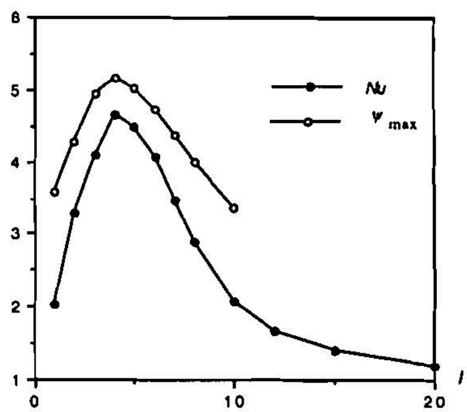


Figure 6. The Nusselt number and extremum value of the stream function versus the aspect ratio for $Ra_v = 1 \times 10^5$.

brational convection at $l \approx 4$. When l is large enough, the intensity of convection decreases as l increases, and we have Nu $\to 1$ and $\psi_{max} \to 0$ at $l \to \infty$. This result is in good agreement with the linear stability theory, that is, in the limiting case of an infinitely long layer with longitudinal temperature gradient and a transversal axis of vibration, the quasi-equilibrium state is possible and it is absolutely stable [3].

CONCLUSIONS

The nonlinear regimes of two-dimensional thermovibrational convection in rectangular cavities subject to a longitudinal temperature gradient and transversal axis of vibration are studied numerically by means of the pseudospectral Chebyshev collocation method. It is shown that (1) the transition from a four-vortices regime to one of inversional symmetry takes place at some critical value of Ra_v, (2) the critical value Ra_{v*} increases monotonously as l increases, (3) the intensity of thermovibrational convection and heat transfer is maximum at $l \approx 4$, (4) in the region $l > 4$ the intensity of convection and heat transfer decreases as l increases, and (5) when $l \gg 4$, the limiting case of rest with conductive heat transfer takes place, in agreement with the theory of stability for the case of an infinitely long fluid layer.

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